I Convolution of two complex Gaussians

The hard core would do this with Laplace transforms but, for the rest of us, here is a complex convolution.

$$
\begin{aligned}
C &= \int_{-\infty}^{\infty} dx_{\Re} \int_{-\infty}^{\infty} dx_{\Im} \, \frac{1}{\pi \upsilon_{\mathfrak{y}}} \exp\left(-\frac{|t-\mathfrak{x}|^2}{\upsilon_{\mathfrak{y}}}\right) \frac{1}{\pi \upsilon_{\mathfrak{x}}} \exp\left(-\frac{|\mathfrak{x}|^2}{\upsilon_{\mathfrak{x}}}\right) && (103)\\
&= \int_{-\infty}^{\infty} dx_{\Re} \int_{-\infty}^{\infty} dx_{\Im} \, \frac{1}{\pi^2 \upsilon_{\mathfrak{y}} \upsilon_{\mathfrak{x}}} \exp\left(-\frac{(t_{\Re}-x_{\Re})^2+(t_{\Im}-x_{\Im})^2}{\upsilon_{\mathfrak{y}}} - \frac{{x_{\Re}}^2+{x_{\Im}}^2}{\upsilon_{\mathfrak{x}}}\right) && (104)\\
&= \int_{-\infty}^{\infty} dx_{\Re} \int_{-\infty}^{\infty} dx_{\Im} \, \frac{1}{\pi^2 \upsilon_{\mathfrak{y}} \upsilon_{\mathfrak{x}}} \exp\left(-\frac{(\upsilon_{\mathfrak{y}}+\upsilon_{\mathfrak{x}})({x_{\Re}}^2+{x_{\Im}}^2) - 2\upsilon_{\mathfrak{x}}(t_{\Re}x_{\Re}+t_{\Im}x_{\Im}) + \upsilon_{\mathfrak{x}}({t_{\Re}}^2+{t_{\Im}}^2)}{\upsilon_{\mathfrak{y}}\upsilon_{\mathfrak{x}}}\right), && (105)\\
&= \int_{-\infty}^{\infty} dx_{\Re} \int_{-\infty}^{\infty} dx_{\Im} \, \frac{1}{\pi^2 \upsilon_{\mathfrak{y}} \upsilon_{\mathfrak{x}}} \exp\left(-E\right). && (106)
\end{aligned}
$$

The trick is to complete the square(s) for $x_{\Re}$ and $x_{\Im}$. The expression inside the exponential becomes

$$
E = \frac{\left(x_{\Re} - \frac{\upsilon_{\mathfrak{x}} t_{\Re}}{\upsilon_{\mathfrak{y}}+\upsilon_{\mathfrak{x}}}\right)^2 + \left(x_{\Im} - \frac{\upsilon_{\mathfrak{x}} t_{\Im}}{\upsilon_{\mathfrak{y}}+\upsilon_{\mathfrak{x}}}\right)^2 - \left(\frac{\upsilon_{\mathfrak{x}} t_{\Re}}{\upsilon_{\mathfrak{y}}+\upsilon_{\mathfrak{x}}}\right)^2 - \left(\frac{\upsilon_{\mathfrak{x}} t_{\Im}}{\upsilon_{\mathfrak{y}}+\upsilon_{\mathfrak{x}}}\right)^2 + \frac{\upsilon_{\mathfrak{x}}({t_{\Re}}^2+{t_{\Im}}^2)}{\upsilon_{\mathfrak{x}}+\upsilon_{\mathfrak{y}}}}{\frac{\upsilon_{\mathfrak{y}}\upsilon_{\mathfrak{x}}}{\upsilon_{\mathfrak{y}}+\upsilon_{\mathfrak{x}}}}, \qquad (107)
$$

and the integrals are Gaussian forms, so

$$
\begin{aligned}
C &= \sqrt{\pi \frac{\upsilon_{\mathfrak{y}}\upsilon_{\mathfrak{x}}}{\upsilon_{\mathfrak{y}}+\upsilon_{\mathfrak{x}}}} \sqrt{\pi \frac{\upsilon_{\mathfrak{y}}\upsilon_{\mathfrak{x}}}{\upsilon_{\mathfrak{y}}+\upsilon_{\mathfrak{x}}}} \frac{1}{\pi^2 \upsilon_{\mathfrak{y}}\upsilon_{\mathfrak{x}}} \exp\left(\frac{(\upsilon_{\mathfrak{x}} t_{\Re})^2}{(\upsilon_{\mathfrak{y}}+\upsilon_{\mathfrak{x}})\upsilon_{\mathfrak{y}}\upsilon_{\mathfrak{x}}} + \frac{(\upsilon_{\mathfrak{x}} t_{\Im})^2}{(\upsilon_{\mathfrak{y}}+\upsilon_{\mathfrak{x}})\upsilon_{\mathfrak{y}}\upsilon_{\mathfrak{x}}} - \frac{\upsilon_{\mathfrak{x}}({t_{\Re}}^2+{t_{\Im}}^2)}{\upsilon_{\mathfrak{y}}\upsilon_{\mathfrak{x}}}\right), && (108)\\
&= \frac{1}{\pi(\upsilon_{\mathfrak{y}}+\upsilon_{\mathfrak{x}})} \exp\left(-\frac{|t|^2}{\upsilon_{\mathfrak{y}}+\upsilon_{\mathfrak{x}}}\right). && (109)
\end{aligned}
$$